

RGE of the Higgs mass, vacuum stability and the hierarchy problem

Li-gong Bian^{1,*}

¹*School of Physics, Nankai University, Tianjin 300071, China*

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Renormalization group equation (RGE) of the Higgs mass up to one-loop order is derived in R_ξ gauge with dimensional regularization method (DREG) base on $\overline{\text{MS}}$ (or $\overline{\text{MS}}$) scheme. The hierarchy problem, which is shown to be gauge independent, survives only in two dimensional not our four dimensional space-time in the sense of DREG. The RGE of the Higgs mass is studied together with renormalization group equation (RGEs) of all interaction couplings of the SM. Scale-dependent properties of the Higgs mass and scalar quartic coupling are investigated, constraints on the upper energy scale limit the SM applied to, given by vacuum stability condition in literatures, is modified slightly.

I. INTRODUCTION

The hierarchy problem is related to fine-tuning problem and problem of naturalness. The hierarchy problem emerges from the fact that the Higgs boson is so much lighter than the Planck mass, to solve this problem, one would expect that the large quantum contributions to the square of the Higgs mass would inevitably make the mass huge, while this calls for an incredible fine-tuning cancellation between the quadratic radiative corrections and the bare mass, thus to make the standard model (SM) “natural”. The famous naturalness problem was studied in ‘t Hooft-Feynman gauge completely three decades before [1]. When proceeding calculations of quadratic divergences, the poles at $d=2$ are kept in DREG, and one correspondence $1/(1-d/2) \rightarrow \Lambda^2/(4\pi)$ was adopted. In order to solve naturalness problem, one need to impose condition $3m_H^2 - 12m_t^2 + 6m_W^2 + 3m_Z^2 = 0$, which is called Veltman condition in some literatures. Gauge bosons are considered as 4-dimensional objects as fermions have four spinors, based on quasi-supersymmetry argument, thus $g_{\mu\nu}g^{\mu\nu} = 4$ instead of $g_{\mu\nu}g^{\mu\nu} = d$, which correspond to dimensional reduction method (DRED) [2, 3], which preserve gauge invariance in d -dimension instead of 4-dimension, and $\text{Tr}[I]=4$ is adopt in this method which side stepped ambiguous of Lorentz indices and Dirac matrices appear in DREG. While only the case $d = 4$ can give our four dimensional physics, thus we consider the quadratic divergences as some physics on the complex two dimensional plane in the sense of [1]. Two decades latter, it was concluded that the Veltman condition is not useful and cannot solve the fine-tuning problem if the scale of New Physics is extremely large [4, 5]. For scale not much larger than the electro-weak scale, it was argue that when $\sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/M_H) \leq v^2/\Lambda^2$ is satisfied, fine-tuning problem will be acceptable [6, 7], where λ_i , M_H , v , Λ are couplings, physics mass of the Higgs boson, and vacuum expectation value (VEV) of the SM and New Physics scale, respectively. One other attempt to solve fine-tuning problem is based on the argument to make fine-tuning more objective [4]. In this paper, we will discuss if this kinds of method to study fine-tuning problem are suitable.

Vacuum stability based on effective potential is always used to impose the upper energy scale where the SM can be applied to [8, 9], which can be achieved by solve RGEs of all couplings, composed of electro-weak couplings and QCD interaction coupling, simultaneously. While we argue that these parameters are not complete enough to express the SM, one more parameter, i.e., VEV or the Higgs mass, is needed. Since the Higgs mass can be observed in experiment, we will choose the Higgs mass as the additional parameter needed, this compose our another motivation. In order to derive RGE of the Higgs mass, we choose DREG to treat divergences and use $\overline{\text{MS}}$ (or $\overline{\text{MS}}$) scheme as renormalization scheme. In order to make our results more universality, all calculations in this work are proceeded in R_ξ gauge. After RGE of the Higgs mass been derived, we investigated scale-dependent properties of the Higgs mass and that of scalar quartic coupling. The renormalization constant of the Higgs mass, wherein poles at $d = 2$ been kept, is also given in this work to derive formula that express the hierarchy problem and to make clear if one can study fine-tuning problem with RGEs obtained by DREG base on $\overline{\text{MS}}$ (or $\overline{\text{MS}}$) scheme.

*Electronic address: lgb@mail.nankai.edu.cn

II. RENORMALIZATION OF THE HIGGS MASS

Direct physical quantities couldn't be gauge dependent, in order to extract some useful physical consequences, we need to explore RGEs of parameters of renormalizable theory in MS (or $\overline{\text{MS}}$) scheme, which has the remarkable property that in this scheme beta functions (β) and anomalous dimension of the mass parameter (γ_m) are gauge-independent. The argument for this is given by [10, 11]. In other renormalization schemes the renormalization coupling constant ($Z_{g,m}$) for coupling or mass is, in general, gauge dependent. This is caused by appearance of the finite terms which dependent on g and ξ , in addition to the terms given in $Z_{g,m}$, with

$$Z_{g,m}(g, \xi) = 1 + \Sigma_{\nu=1} \frac{Z_{g,m}^{(\nu)}(g, \xi)}{\varepsilon^\nu}, \quad (1)$$

where g, ξ are renormalized couplings and gauge parameters, respectively.

One may note that no explicit scale parameter μ -dependent in Eq. (1), so RGEs of couplings (beta functions) and anomalous dimension of mass operator γ_m in MS (or $\overline{\text{MS}}$) scheme, which are functions of $Z_{g,m}$, carry no explicit μ -dependent, thus MS (or $\overline{\text{MS}}$) is always referred as the mass-independent renormalization scheme. This property of MS (or $\overline{\text{MS}}$) scheme make it very easy to solve RGEs.

A. The Lagrangian and counter-term method

First of all, notations on relationships between renormalized masses and parameters used in this work are

$$\begin{aligned} m_H &= \sqrt{2\lambda} v, & m_W &= \frac{g_2 v}{2}, & m_Z &= \frac{g_1 v}{2\cos\theta_W}, \\ m_t &= \frac{g_t v}{\sqrt{2}}, & \cos\theta_W &= \frac{g_2}{\sqrt{g_2^2 + g_1^2}}. \end{aligned} \quad (2)$$

with λ, g_t, g_2 and g_1 are scalar quartic coupling, top-quark Yukawa coupling, $\text{SU}(2)_L$ and $\text{U}(1)$ gauge couplings, respectively.

After SSB, bare Lagrangian of the Higgs part in the SM is

$$\begin{aligned} \mathcal{L}_H^0 &= \frac{1}{2}(\partial_\mu H^0)^2 - \frac{1}{2}(m_H^0)^2(H^0)^2 - \lambda^0 v^0(H^0)^3 - \frac{1}{4}\lambda^0(H^0)^4 \\ &\quad + \text{const.}, \end{aligned} \quad (3)$$

where superscripts “0” on mass, couplings and the Higgs field are used to denote that parameters are bare parameters, parameters which do not have superscripts represent renormalized parameters of the SM, and $m_H^0 = \sqrt{2\lambda^0}v^0$ have been sat when the above equation is written down. Let us first introduce four renormalization constants to relate bare and renormalized parameters,

$$\lambda^0 = Z_H^{-2} Z_1 \lambda, \quad (m_H^0)^2 = Z_H^{-1} Z_0 (m_H)^2, \quad H^0 = Z_H^{1/2} H. \quad (4)$$

Then, relationship between bare and renormalized vacuum energy (VEV) is given by $v^0 = Z_1^{-1/2} Z_H^{1/2} Z_0^{1/2} v$, the bare Lagrangian of the Higgs part \mathcal{L}_H^0 can be separated to renormalized part \mathcal{L}_H and counter-term Lagrangian \mathcal{L}_H^{ct} , with \mathcal{L}_H precisely equal to \mathcal{L}_H^0 if bare parameters in \mathcal{L}_H^0 are replaced by the renormalized ones, and

$$\mathcal{L}_H^{ct} = \frac{1}{2}(Z_H - 1)(\partial_\mu H)^2 - \frac{1}{2}(Z_0 - 1)(m_H)^2 H^2 - (Z_1^{1/2} Z_0^{1/2} - 1)\lambda v H^3 - \frac{1}{4}(Z_1 - 1)\lambda H^4 + \text{const.}, \quad (5)$$

with renormalization constant for the Higgs mass $Z_m = Z_H^{-1} Z_0$ and scalar quartic coupling renormalization constant $Z_\lambda = Z_H^{-2} Z_1$, then one arbitrary mass parameter μ can be introduced through $\lambda^0 = Z_\lambda \lambda \mu^\varepsilon$, similarly, scale μ can also be introduced through $(g^0)_{1,2,t}^2 = Z_{g_1, g_2, g_t} g_{1,2,t}^2 \mu^\varepsilon$, with $\varepsilon = 4 - d$. Since beta function of scalar quartic coupling (β_λ) have been derived decades before [12–14], we will not derive renormalization constant Z_1 anymore, but use β_λ directly. And since VEV is experimental unobservable, RGE of VEV will not been explored.

In MS (or $\overline{\text{MS}}$) scheme, divergent terms in self-energy of the Higgs field will be subtracted by renormalization constant. Two-point self-energy of the Higgs field comes from 1PI self-energy and tadpole contributions,

$$\Sigma_H(p^2) = \Sigma_H^{1\text{PI}}(p^2) + \Sigma_H^T(p^2) \quad (6)$$

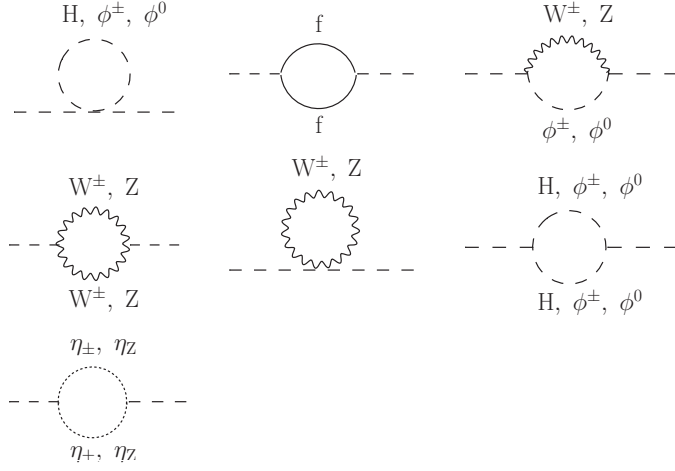


FIG. 1: One-loop 1PI self-energy corrections to the Higgs mass

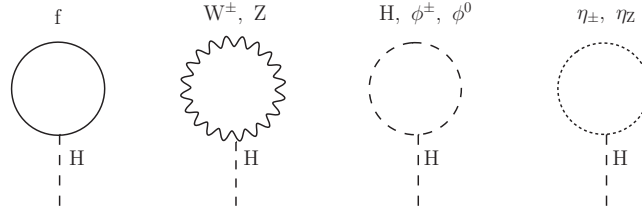


FIG. 2: Tadpole Feynman diagram with one external Higgs field in the SM

with Feynman diagrams contribute to 1PI self-energy shown in Fig. 1, and the tadpole diagrams contribute to the Higgs self-energy is

$$\Sigma_H^T = -i \frac{3(m_H)^2}{v} \frac{i}{-(m_H)^2} T \quad (7)$$

where $\frac{i}{-(m_H)^2}$ is the propagator of the Higgs boson carrying *zero* momentum, the Higgs three-point vertex is $\frac{-i3(m_H)^2}{v}$, with “T” represent Feynman diagrams contributions shown in Fig. 2.

Up to one-loop level, the counter-term method requires $\Sigma_H(p^2) + i(Z_H - 1)p^2 - i(Z_0 - 1)m_H^2 = 0$, combined with the second formula given in Eq. (4) and the relationship $Z_m = Z_H^{-1}Z_0$, we can derive renormalization constant of the Higgs mass (Z_m).

B. Renormalization coupling constant Z_m and anomalous dimension of the Higgs mass γ_{m_H}

Scalar momentum integral which can give rise to quadratic divergences involved in self-energy calculations are shown below and other momentum integrals are listed in section. V

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2} = -i \frac{1}{4\pi} \frac{1}{1 - d/2} + i \frac{m^2}{2 - d/2} \quad (8)$$

where two poles $1/(1 - d/2)$ and $1/(2 - d/2)$ are all kept, while when one computes the momentum integration shown in Eq. (8), one can expand results around $d = 2$ or $d = 4$, which give two different poles, when dimension d continued to “4”, corresponding to four dimension physics, then there is no pole at $d = 2$, which give quadratic divergences on the complex two dimensional plane in the sense of [1].

Proceeding calculations in DREG and keeping poles $1/(2 - d/2)$, renormalization constant of the Higgs field can be calculated,

$$Z_H = 1 - \frac{1}{(4\pi)^2} \frac{1}{2 - d/2} \left[\frac{g_2^2(\xi - 3)}{2} + \frac{g_1^2 + g_2^2}{4}(\xi - 3) + 3g_t^2 \right], \quad (9)$$

and renormalization constant is derived as:

$$Z_0 = 1 + \frac{1}{(4\pi)^2} \frac{1}{2-d/2} \left(6\lambda - \frac{3\xi}{4} g_2^2 - \frac{\xi}{4} g_1^2 \right). \quad (10)$$

Thus, renormalization constant of the Higgs mass on the basis of MS (or $\overline{\text{MS}}$) scheme is calculated to be

$$\begin{aligned} Z_m &= Z_H^{-1} Z_0 \\ &= 1 + \frac{1}{(4\pi)^2} \frac{2}{4-d} \left(6\lambda + 3g_t^2 - \frac{9}{4} g_2^2 - \frac{3}{4} g_1^2 \right), \end{aligned} \quad (11)$$

it is obviously that Z_m is gauge independent.

RGE of the Higgs mass in MS (or $\overline{\text{MS}}$) can be given by

$$\mu \frac{dm_H}{d\mu} = -m_H \lim_{\varepsilon \rightarrow 0} \gamma_{m_H}(m_H(\mu), \varepsilon), \quad (12)$$

with

$$\begin{aligned} \gamma_{m_H}(m_H(\mu), \varepsilon) &= \frac{\mu}{2Z_{m_H}} \frac{dZ_m}{d\mu} \\ &= \frac{\mu}{2Z_m} \left(\frac{\partial Z_m}{\partial \lambda} \beta(\lambda(\mu), \varepsilon) + \frac{\partial Z_m}{\partial g_t} \beta(g_t(\mu), \varepsilon) + \frac{\partial Z_m}{\partial g_1} \beta(g_1(\mu), \varepsilon) + \frac{\partial Z_m}{\partial g_2} \beta(g_2(\mu), \varepsilon) \right) \end{aligned} \quad (13)$$

represent anomalous dimension of the Higgs mass term, wherein

$$\beta(\lambda(\mu), \varepsilon) = -\varepsilon \lambda + \beta_\lambda = \mu \frac{d\lambda(\mu)}{d\mu}, \quad (14)$$

$$\beta(g_t(\mu), \varepsilon) = -\frac{\varepsilon}{2} g_t + \beta_{g_t} = \mu \frac{dg_t(\mu)}{d\mu}, \quad (15)$$

$$\beta(g_1(\mu), \varepsilon) = -\frac{\varepsilon}{2} g_1 + \beta_{g_1} = \mu \frac{dg_1(\mu)}{d\mu}, \quad (16)$$

$$\beta(g_2(\mu), \varepsilon) = -\frac{\varepsilon}{2} g_2 + \beta_{g_2} = \mu \frac{dg_2(\mu)}{d\mu}, \quad (17)$$

are RGEs of λ , g_t , g_1 , g_2 (beta functions), the additional factor “ $\frac{1}{2}$ ” before $g_{t,1,2}$ in the last three equations compare with the first equation above is caused by the introduction approach of μ , as described below Eq. (5), and they reduce to beta functions: β_λ , β_{g_t} , β_{g_1} , β_{g_2} in four dimension.

Since the anomalous dimension of the Higgs mass (γ_{m_H}) is functions of Z_m , it must be gauge independent. While in other renormalization schemes the renormalization coupling constant and m_H are, in general, gauge dependent. This is caused by appearance of the finite terms in addition to the terms given in Z_m at the right side of Eq. (11) in other schemes.

Up to one-loop level, RGE of the Higgs mass in $\overline{\text{MS}}$ scheme in four dimension is given by

$$\mu \frac{dm_H}{d\mu} = -m_H \gamma_{m_H}, \quad (18)$$

with anomalous dimension of the Higgs mass

$$\gamma_{m_H} = -\frac{1}{16\pi^2} \left(6\lambda + 3g_t^2 - \frac{9}{4} g_2^2 - \frac{3}{4} g_1^2 \right). \quad (19)$$

Here one need to note that others Yukawa couplings except of that of top quark are all left out for small contributions.

While when we proceed calculations, we keep all poles at $d = 2$ and $d = 4$ for completeness, renormalization constant for the Higgs field Z_H will be the same with Eq. (9), while renormalization constant Z_0 include contributions from terms that represent poles at $d = 2$,

$$Z'_0 = 1 + \frac{2}{(4\pi)m_H^2} \frac{1}{1-d/2} \left[6\lambda - \frac{3}{2} \text{Tr}[I] g_t^2 + (g_\mu^\mu - 1) \left(\frac{3g_2^2}{4} + \frac{g_1^2}{4} \right) \right] + \frac{1}{(4\pi)^2} \frac{1}{2-d/2} \left(6\lambda - \frac{3\xi}{4} g_2^2 - \frac{\xi}{4} g_1^2 \right). \quad (20)$$

Now, the renormalization constant of the Higgs mass is

$$\begin{aligned} Z'_m &= Z_H^{-1} Z'_0 \\ &= 1 + \frac{2}{(4\pi)(m_H)^2} \frac{1}{1-d/2} \left[6\lambda - \frac{3}{2} \text{Tr}[I] g_t^2 + (g_\mu^2 - 1) \left(\frac{3g_2^2}{4} + \frac{g_1^2}{4} \right) \right] + \frac{1}{(4\pi)^2} \frac{2}{4-d} \left(6\lambda + 3g_t^2 - \frac{9}{4} g_2^2 - \frac{3}{4} g_1^2 \right) \end{aligned} \quad (21)$$

Use Eq. (12) base on MS (or $\overline{\text{MS}}$) scheme, one can derive anomalous dimension of the Higgs mass, γ'_{m_H} . While since when one proceed calculations of renormalization constant of couplings (λ , g_t , g_1 , g_2 and g_3) in the SM, no poles at $d = 2$ appear in four dimension, so poles at $d = 2$ do not contribute to beta functions in four dimension.

And from Eq. (21), one may found that if we take replacement $1/(1-d/2) \rightarrow \Lambda^2/(4\pi)$, which can be obtained when one compare Eq. (8) with the same integral that calculated with naive cut-off method, and consider the pole at $d = 2$ alone, from the fact that dimension d couldn't compacted to "2" and "4" at the same time in the sense of Eq. (8), then the hierarchy problem can be expressed by

$$(m_H^0)^2 = (m_H)^2 + \frac{2\Lambda^2}{(4\pi)^2 v^2} [3(m_H)^2 - 12(m_t)^2 + 6(m_W)^2 + 3(m_Z)^2]. \quad (22)$$

To get this formula, $\text{Tr}[I]=g_\mu^2=4$ and Eq. (2) need to be adopt in the second term at right side of Eq. (21).

Here one can also argue that the so called hieray problem is the appearance of physics on the complex two dimensional plane. Further more it have been identified that the Veltman condition wouldn't be true [7] up to two loop level, we argue that this is caused by the fact that the physics of four dimensional won't be mixed with those of two dimensional. The leading divergences at two-loop order is $1/((1-d/2)(2-d/2))$ [15], as can be seen from the last formula in section. V.

Based on above arguments, any attempt to analyse fine-tuning with RGEs of the SM which applied to four dimension is not appropriated. If one want to explore fine-tuning problem in the SM, RGEs that can be applied to two dimension are needed.

III. SCALE-DEPENDENT PROPERTY OF THE SCALAR QUARTIC COUPLING AND THE HIGGS MASS OF THE SM

Since only when all RGEs in the SM been studied together, can we investigate physics in the system of the SM, i.e., all electro-weak couplings (λ , g_1 , g_2 , g_t) and QCD couplings g_3 and the Higgs mass together compose the whole physical system of the SM. To study the vacuum stability problem of the SM, we need also explore scale dependent property of the Higgs mass except of that of RGEs of all couplings, while the RGE of the Higgs mass is omitted in literatures [9]. To explore the μ -dependent property of the Higgs mass, all RGEs of all couplings of the SM are needed.

A. Scalar quartic coupling in the SM

The absolute vacuum stability of the SM requires the scalar quartic coupling $\lambda(\mu) > 0$, called vacuum stability condition in literatures [9], which can be used to fix the energy scale Λ where new physics enter. When we analyse property of $\lambda(\mu)$ in the SM without considering RGE of the Higgs mass, the behavior of $\lambda(\mu)$ is plotted in Fig. 3, where β functions of couplings in the SM are considered up to two-loop order. When we solve beta functions of all couplings up to two-loop order [13, 14], the matching conditions given in [8, 16, 17], i.e., matching of $\overline{\text{MS}}$ coupling constants and pole masses to give boundary conditions of couplings up to two-loop order, are used. From Fig. 3, we find that, up to two-loop level, depicted by the dashed line, the energy scales where new physics enter is about $\Lambda = 10^{10}$ GeV. We also plot the behaviour of λ with respect to μ at one-loop order for compare convenience with the result when RGE of the Higgs mass is considered, as given following, and the boundary conditions of RGEs of couplings at one-loop order are the same with the scenario given below.

When we solve RGEs of all couplings and the Higgs mass up to one-loop order at the same time, boundary conditions of m_H and g_t are set to $m_H(126 \text{ GeV}) = 126 \text{ GeV}$ and $g_t(M_t) = \sqrt{2}M_t/v$, with M_t is pole mass of top quark [18] and VEV $v=246 \text{ GeV}$, and boundary conditions of g_1 and g_2 can be obtained as in [8, 19]. The scalar quartic coupling behaviours in high-energy regions is plotted in Fig. 4. From Fig. 4, we can find that viable region of the SM is $\mu \leq 10^5 \text{ GeV}$. The constraint $\mu \leq 10^5 \text{ GeV}$ is given by the vacuum stability condition, i.e., the scalar quartic coupling need to be not smaller than *zero*. Thus when RGE of m_H is considered together with all beta functions of couplings in the SM, comparing Fig. 4 with the one-loop case shown in Fig. 3, we find that the bound on upper energy scale, where the SM can applied to, is extended from $10^{3\cdots 4} \text{ GeV}$ to about 10^5 GeV .

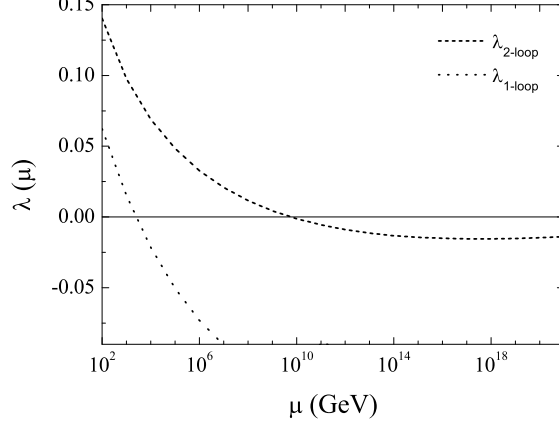


FIG. 3: Scalar quartic coupling behavior

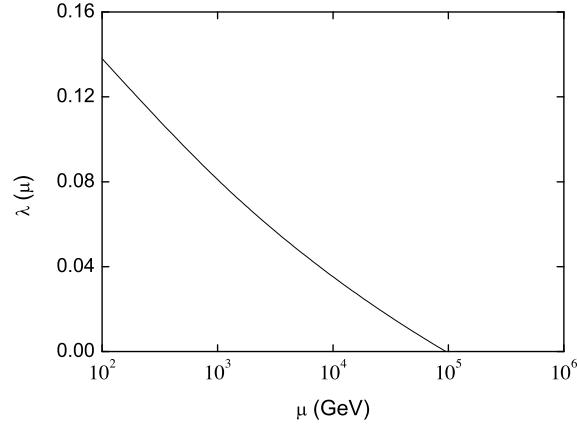


FIG. 4: Scalar quartic coupling in low energy region

B. Scale-dependent property of the Higgs mass

Now, we will turn to analyse the scale-dependent property of the Higgs mass. The beta function for a generic coupling X is given as:

$$\mu \frac{dX}{d\mu} = \beta_X = \frac{\beta_X}{16\pi^2}. \quad (23)$$

The list of beta functions up to one-loop order are given below [7, 13]:

$$\beta_\lambda = \lambda(-9g_2^2 - 3g_1^2 + 12g_t^2) + 24\lambda^2 + \frac{3}{4}g_2^4 + \frac{3}{8}(g_1^2 + g_2^2)^2 - 6g_t^4, \quad (24)$$

$$\beta_{g_t} = \frac{9}{2}g_t^3 + g_t \left(-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right), \quad (25)$$

$$\beta_{g_1} = \frac{41}{6}g_1^3, \quad \beta_{g_2} = -\frac{19}{6}g_2^3, \quad \beta_{g_3} = -7g_3^3. \quad (26)$$

Solving Eq. (18) together with Eq. (23) numerically, behavior of the Higgs mass with respect to scale μ in high-energy region is plotted in Fig. 5. We can find that the Higgs mass growing and then damping from about $\mu=10^8$ GeV with the scale μ growing and always larger than *zero*.

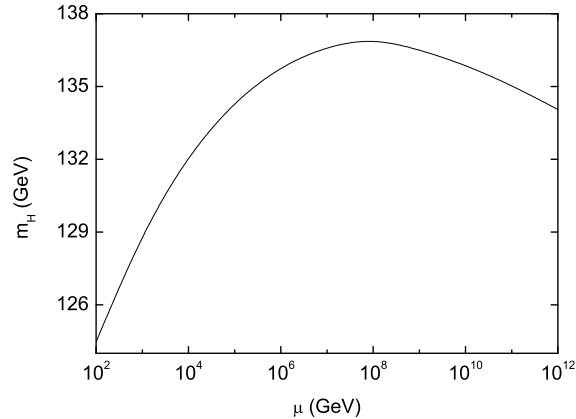


FIG. 5: Behavior of the Higgs mass in high energy region

From the viewpoint of the Higgs potential [9, 20], when the Higgs field is not far larger than the mass term with the “wrong” sign, wherein $V(H)$ composed of $-\frac{m^2}{2}H^2 + \frac{\lambda}{4}H^4$ not only $\frac{\lambda}{4}H^4$, the vacuum stability condition need to be changed slightly, the requirement $\lambda(\mu) > 0$ is not enough. The stability condition requires $V(H) > 0$ then, which give rise to $H^2 < 2v^2$. The nonnegative Higgs mass-square means that $d^2V(H)/dH^2 > 0$, which imply that $H^2 < v^2/3$, with v given by condition $dV(H)dH|_{H=v} = 0$. While consider the Fig. 5 together with Fig. 4, one may find that $v^2 < 0$ when the energy scale is above 10^5 GeV. This turns the Higgs field to be an complex field. Combine these two bounds on the value of Higgs fields give to $H^2 < 2v^2$.

IV. CONCLUSION

In this paper, RGE of the Higgs mass is derived at one-loop level, with DREG base on MS (or $\overline{\text{MS}}$) scheme in R_ξ gauge, which is gauge independent as required for. The so called hierarchy problem is derived also in R_ξ , which may be the physics on the complex two dimensional plane. Any attempt to do numerical analysis on fine-tuning with beta functions derived in DREG method based on MS (or $\overline{\text{MS}}$) scheme in four dimensional theory is not suitable, for the beta functions apply to four dimension not two dimension. The study on vacuum stability based on effective potential in literatures are not appropriate enough, since the RGE of m_H need to be investigate together with other RGEs of couplings of the SM, simultaneously. Based on this argument, the scale-dependent properties of m_H and $\lambda(\mu)$ are studied, we find that, in the case the value of the Higgs field is far large than the mass term with the “wrong” sign, with the energy scale become higher and higher, m_H become bigger and bigger smoothly within the valid energy region of the SM, i.e., $\mu \leq 10^5$ GeV, which is given by the requirement that the vacuum should be stable at one-loop order. While when the Higgs field is not far larger than m , the vacuum stability requires $H^2 < 2v^2$, with $v^2 < 0$, then the Higgs field change to be one complex field.

V. APPENDIX: INTEGRATION FORMULA INVOLVED IN DIVERGENCES CALCULATION

Scalar and tensor integrals involved in one-loop calculations,

$$\begin{aligned}
\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)^2} &= \frac{i}{(4\pi)^2} \frac{1}{2 - d/2}, \\
\int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - m^2)^2} &= -\frac{g^{\mu\nu}}{2} \frac{i}{4\pi} \frac{1}{1 - d/2} + i \frac{2g^{\mu\nu}}{4} \frac{m^2}{(4\pi)^2} \frac{1}{2 - d/2}, \\
\int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - m^2)^3} &= \frac{ig^{\mu\nu}}{4(4\pi)^2} \frac{1}{2 - d/2}, \\
\int \frac{d^d k}{(2\pi)^d} \frac{k^4}{(k^2 - m^2)^3} &= -\frac{i}{4\pi} \frac{1}{1 - d/2} + i3 \frac{m^2}{(4\pi)^2} \frac{1}{2 - d/2}, \\
\int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\sigma k^\nu k^\rho}{(k^2 - m^2)^4} &= \frac{i}{24(4\pi)^2} (g^{\mu\sigma} g^{\nu\rho} + g^{\mu\nu} g^{\sigma\rho} + g^{\mu\rho} g^{\sigma\nu}) \frac{1}{2 - d/2}.
\end{aligned} \tag{27}$$

Scalar integral that give rise to the leading divergences at two-loop order,

$$\begin{aligned}
&\int \frac{d^d k d^d p}{(2\pi)^d (2\pi)^d} \frac{1}{(k^2 - m_1^2)(p^2 - m_2^2)((k - p)^2 - m_3^2)} \\
&= \frac{1}{(4\pi)^3} \frac{1}{1 - d/2} \frac{1}{2 - d/2}.
\end{aligned} \tag{28}$$

Poles at $d=2$ and $d=4$ are all kept in above momentum integrals, though only one of them can survive at the same time.

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